

General Certificate of Education
January 2008
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Thursday 24 January 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Given that $\frac{3}{9-x^2}$ can be expressed in the form $k\left(\frac{1}{3+x} + \frac{1}{3-x}\right)$, find the value of the rational number k . (2 marks)
- (b) Show that $\int_1^2 \frac{3}{9-x^2} dx = \frac{1}{2} \ln\left(\frac{a}{b}\right)$, where a and b are integers. (3 marks)
- 2 (a) The polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 18x + 8$.
- (i) Use the Factor Theorem to show that $(2x - 1)$ is a factor of $f(x)$. (2 marks)
- (ii) Write $f(x)$ in the form $(2x - 1)(x^2 + px + q)$, where p and q are integers. (2 marks)
- (iii) Simplify the algebraic fraction $\frac{4x^2 + 16x}{2x^3 + 3x^2 - 18x + 8}$. (2 marks)
- (b) Express the algebraic fraction $\frac{2x^2}{(x+5)(x-3)}$ in the form $A + \frac{B+Cx}{(x+5)(x-3)}$, where A , B and C are integers. (4 marks)
- 3 (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in x^2 . (2 marks)
- (b) Hence obtain the binomial expansion of $\sqrt{1 + \frac{3}{2}x}$ up to and including the term in x^2 . (2 marks)
- (c) Hence show that $\sqrt{\frac{2+3x}{8}} \approx a + bx + cx^2$ for small values of x , where a , b and c are constants to be found. (2 marks)

- 4 David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price, £ P , of this house, where t is the time in years after 1 January 1885 and A and k are constants.

- (a) (i) Write down the value of A . (1 mark)
- (ii) Show that, to six decimal places, $k = 1.079775$. (2 marks)
- (iii) Use the model, with this value of k , to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. (2 marks)
- (b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price, £ Q , of this house t years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price. (4 marks)

- 5 A curve is defined by the parametric equations $x = 2t + \frac{1}{t^2}$, $y = 2t - \frac{1}{t^2}$.

- (a) At the point P on the curve, $t = \frac{1}{2}$.
- (i) Find the coordinates of P . (2 marks)
- (ii) Find an equation of the tangent to the curve at P . (5 marks)
- (b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where k is an integer. (3 marks)

Turn over for the next question

Turn over ►

6 A curve has equation $3xy - 2y^2 = 4$.

Find the gradient of the curve at the point $(2, 1)$.

(5 marks)

7 (a) (i) Express $6 \sin \theta + 8 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give your value for α to the nearest 0.1° .

(2 marks)

(ii) Hence solve the equation $6 \sin 2x + 8 \cos 2x = 7$, giving all solutions to the nearest 0.1° in the interval $0^\circ < x < 360^\circ$.

(4 marks)

(b) (i) Prove the identity $\frac{\sin 2x}{1 - \cos 2x} = \frac{1}{\tan x}$.

(4 marks)

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0^\circ < x < 360^\circ$.

(4 marks)

8 Solve the differential equation

$$\frac{dy}{dx} = \frac{3 \cos 3x}{y}$$

given that $y = 2$ when $x = \frac{\pi}{2}$. Give your answer in the form $y^2 = f(x)$.

(5 marks)

9 The points A and B lie on the line l_1 and have coordinates $(2, 5, 1)$ and $(4, 1, -2)$ respectively.

(a) (i) Find the vector \overrightarrow{AB} .

(2 marks)

(ii) Find a vector equation of the line l_1 , with parameter λ .

(1 mark)

(b) The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on l_2 .

(2 marks)

(ii) The point Q lies on l_1 and is such that PQ is perpendicular to l_2 . Find the coordinates of Q .

(6 marks)

END OF QUESTIONS